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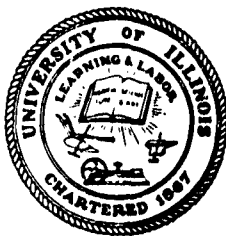
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ANALYSIS OF MISSILE LAUNCHERS
Part $_R$

Approximate Formulas for Pitch Velocity at Tip-off

by

M. Stippes

DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS
UNIVERSITY OF ILLINOIS

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A Research Project of the

Department of Theoretical and Applied Mechanics

University of Illinois

Sponsored by

University of Illinois
Urbana, Illinois
August, 1962

This is a part of the report on University of Illinois Project No. 46-22-60-304, "Launcher Dynamics Study". The analysis was performed under Contract No. DA-11-070-508-ORD-593 (ORD Proj. No. 517-01-002, CMS CODE 55-30.11.577). In part, the contract concerns the study of possible types of models for the representation of launching systems. This particular report contains approximate formulas for estimating pitch velocity and its changes at tip-off due to thrust misalignment, SOSR effects, friction, and rail curvature.

The program is under the technical supervision of Rock Island Arsenal (RIA), Rock Island Illinois, and the administrative supervision of Chicago Ordnance District.

Respectfully submitted
University of Illinois

M. Stippes, Project Supervisor
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        Applied Mechanics

LAUNCHER DYNAMICS STUDY

Part R

Approximate Formulas for Pitch Velocity at Tip-off

ABSTRACT:

The preliminary design or feasibility study of a straight rail launching system requires, in addition to layouts and material composition, estimates of the accuracy of the system. Possible sources of inaccuracy, which is defined to be nonreproducable tip-off velocity, are thrust misalignment, rail curvature arising from manufacturing processes or uneven heating due to the sun, shaking forces of unknown phase which might exist in SOSR designs, or excessive amounts of friction between shoes and rail.

Our purpose in this report is to present techniques of analysis for a straight rail system with a view toward obtaining estimates of changes in pitch velocity of the rocket at tip-off as might exist because of the presence of these various disturbing agencies.

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LIST OF SYMBOLS

ξ, η	coordinates attached to launcher
ξ_m, η_m	coordinates of mass center of missile
θ	angular rotation of launcher from static position
γ	angle of elevation in absence of dead weight
θ_{st}	angular rotation due to dead weight
β	helical spring constant
a, b, c, e, f, η_o	linear dimensions of launcher and missile
W_L	weight of launcher
W_m	weight of missile
$\epsilon\, f(\xi)$	function specifying curvature of the rail
ψ	additional angle of inclination of missile due to rail curvature
T	thrust
δ, ϵ_2	thrust misalignment parameters
ϵ_1, I_{xy}, λ_1, λ_2	mass unbalance parameters
Ω	angular frequency of shaking force
μ	coefficient of friction
P, Q	shoe loads
I_L	moment of inertia of launcher about pivot
I_m	moment of inertia of missile about mass center

Additional symbols are defined in the body of the text.

I. INTRODUCTION

The preliminary design or feasibility study of a straight rail launching system requires, in addition to layouts and material composition, estimates of its accuracy. Possible sources of inaccuracy, which is defined to be nonreproducable tip-off pitch velocity, are thrust misalignment, rail curvature arising from manufacturing processes or uneven heating due to the sun, shaking forces which might exist in SOSR designs, or excessive amounts of friction between shoes and rail.

Our purpose in this report is to present techniques of analysis for a straight rail system with a view toward obtaining estimates of changes in pitch velocity of the missile at tip-off as might exist because of the presence of these various disturbing agencies. Since these results are to be used for feasibility studies or the like, methods will be developed so that these quantities can be obtained by means of slide rule or desk machine calculations rather than by means of analogue or digital computing devices. For the detailed analyses attendent to final design proposals, numerical investigations of the type carried out in many of the previous technical reports of this project are necessary.

All of these methods are associated with a launcher whose gross configuration can be accurately represented by the two degree of freedom model which appeared in TAM Report No. 552, University of Illinois, Urbana, Illinois, 1960. As shown in Fig. 1 , this model consists of a rigid L-frame supported at a single point O . Since this is a pivot point, horizontal and vertical motion is prevented and so only rotation is possible. Stability is provided by some equivalent helical spring which represents the overall flexibility of the structure. The launching rail AB may have some small curvature as indicated.
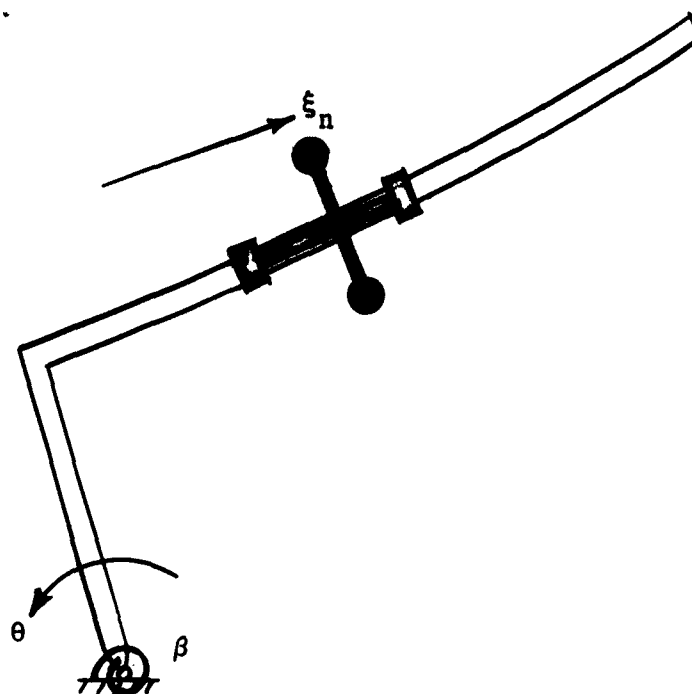
FIGURE 1

The missile is represented by two masses rigidly attached to two sleeves that slide along the arm of the launching frame. These masses are the source of rotational inertia for the missile. (We only consider systems in which the shoes of the rocket leave the rail simultaneously.) Finally the thrust force is denoted by $T(t)$.

This dynamical system has two degrees of freedom. The coordinates used to describe its configuration at time t are the angular rotation $\theta(t)$ of its arm as measured from equilibrium and $\xi_m(t)$, the distance the mass center of the missile has moved up the rail. As in all previous work, it is again assumed that the angle θ is small so that the usual approximations based on linearization are permissible.

The differential equations which describe the motion of this system are very complex. We may; however, in certain instances, bypass complete integration of the equations of motion of any system. Our efforts here are based on the reduction of the differential equations of motion to integral equations and a subsequent solution by iteration. The necessary number of steps in the iteration depends on the numerical values of a parameter which is the ratio launch time versus natural period of the system. In some instances we can employ the principle of impulse and momentum to interpret these approximations.

Equations of Motion for the Model

We begin with a discussion of the kinematics of the model shown schematically in Fig. 2 . The missile is represented by the bent ABCD and the launcher by the rigid frame ORS . Three coordinate systems are introduced. The first of these labelled XY in the figure is fixed to ground, OX being in the tangent plane to the earth in the direction of aim. A second system denoted by xy is also fixed and in the XY-plane; however, the x-axis of this set of coordinates makes an angle $\gamma + \theta_{st}$ with X . This total angle is the angle of elevation when the structure is loaded with its dead weight. Finally the $\xi\eta$-system is attached to the launcher and rotates with it. This angle of rotation which will be called θ represents the deviation of the launcher from its equilibrium position due to its weight.

The rail RS , has a slight curvature. Its equation in the $\xi\eta$-system will be taken in the form
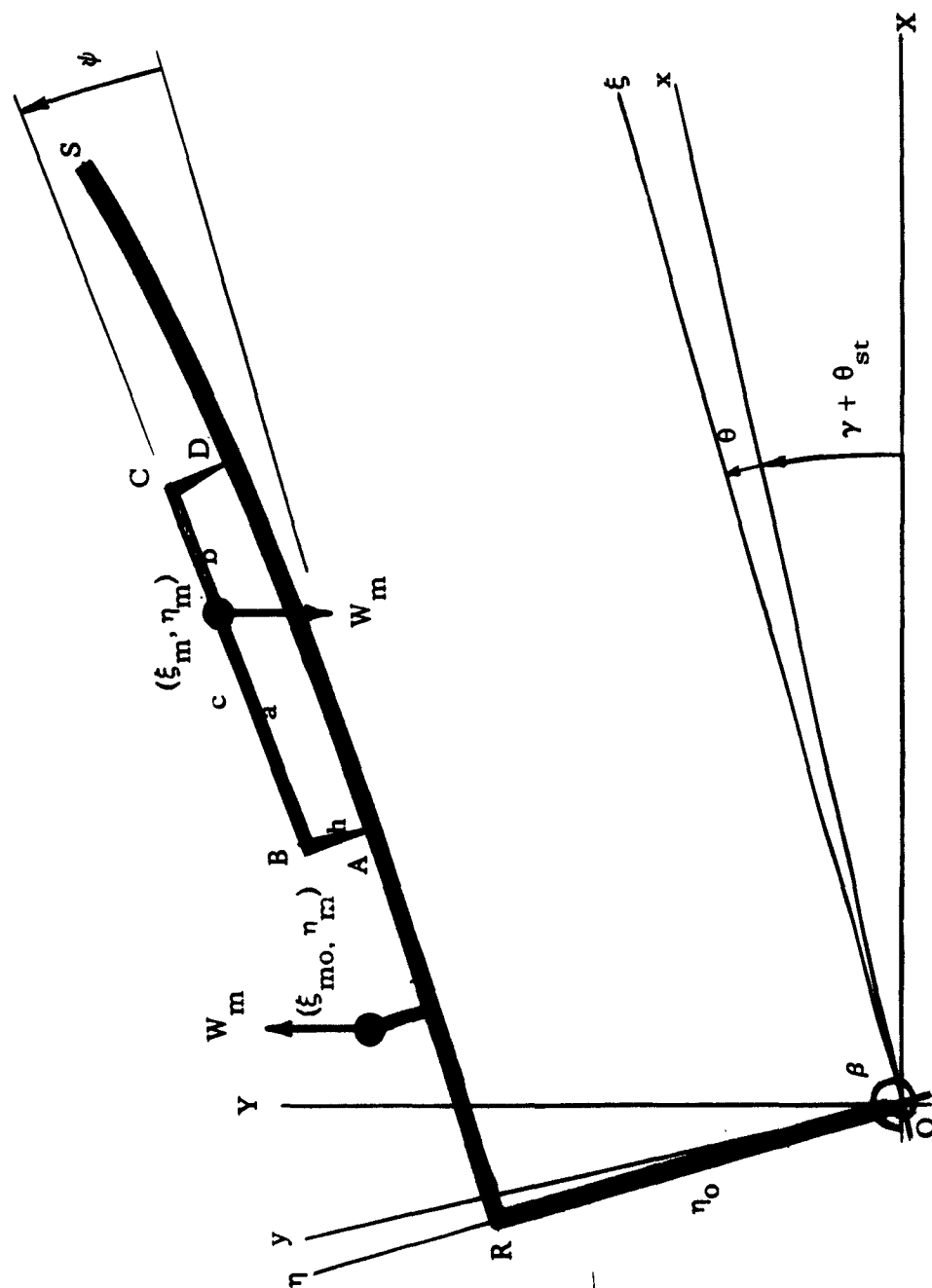
$$\eta_{rail} = \eta_0 + \epsilon \, f(\xi)$$

FIGURE 2

where ϵ is small and of the order of magnitude of the angle θ. If (ξ_m, η_m) are the coordinates of the mass center of the rocket in the $\xi\eta$-system, the angle of rotation ψ of the missile relative to this set of coordinates is given by

$$\psi = \epsilon \frac{f(\xi_m + b) - f(\xi_m - a)}{c}.$$

To obtain the equations of motion for the system we first consider the free body diagram of the missile shown in Fig. (3). These equations will be established relative to the $\xi\eta$-coordinates. The following forces act on the rocket:

$T(t)$ - the thrust force with misalignment parameters ϵ_2 and δ,

$\dfrac{W_m}{g} \epsilon_1 \Omega^2 \sin(\Omega t + \lambda_2)$ - shaking force due to mass unbalance in

SOSR, rocket,

$I_{xy} \Omega^2 \sin(\Omega t + \lambda_1)$ - shaking moment due to mass unbalance in

SOSR rocket,

Q, P - front and rear shoe loads,

$\mu |Q|, \mu |P|$ - friction forces at the shoes.

All of the quantities $\epsilon_1, \epsilon_2, \delta, I_{xy}$ are assumed to be small so that we may neglect products of these with θ, ψ, or ϵ.

For the motion of the mass center of the missile we have

$$m\, a_\xi = F_\xi:$$

$$\frac{W_m}{g}\left[\ddot{\xi}_m - (\eta_0 + h)\ddot{\theta}\right] = T - W_m \sin\gamma$$

$$- \mu |P| - \mu |Q| - P\epsilon f'(\xi_m - a) - Q\epsilon f'(\xi_m + b),$$

(1)

FIGURE 3

$m \, a_\eta = F_\eta$:

$$\frac{W_m}{g} \left[\ddot{\theta} \, \xi_m + 2 \dot{\xi}_m \, \dot{\theta} + \epsilon \, \ddot{f}(\xi_m - a) + a \, \ddot{\psi} \right] = P + Q$$

$$- \mu \, \epsilon \left[|P| \, f'(\xi_m - a) + |Q| \, f'(\xi_m + b) \right] - W_m \cos \gamma$$

$$+ \frac{W_m}{g} \, \epsilon_1 \, \Omega^2 \sin(\Omega t + \lambda_2) + T(\delta + \psi). \tag{2}$$

The angular motion of the rocket is expressed in the following relation

$$I_m(\ddot{\theta} + \ddot{\psi}) = Q \, b - Q \, \epsilon \, h \, f'(\xi_m + b) - \mu \, |Q| \, h - \mu \, |Q| \, \epsilon \, f'(\xi_m + b) \, b$$

$$- P \, a - P \epsilon \, f'(\xi_m - a) \, h - \mu \, |P| \, h + \mu \, |P| \, \epsilon \, f'(\xi_m - a) \, a - T \, \epsilon_2 - T \delta (a + d)$$

$$+ I_{xy} \, \Omega^2 \sin(\Omega t + \lambda_1). \tag{3}$$

These equations contain the unknowns θ, ξ_m, P, Q. One final equation is sufficient for a unique determination of these variables. The moment of momentum principle with respect to the point O for the entire system will give one such equation:

$$\frac{d}{dt} \left\{ I_L \, \dot{\theta} + I_m (\dot{\theta} + \dot{\psi}) - \frac{W_m}{g} \left[\dot{\xi}_m \left(\eta_0 + h + \epsilon \, f(\xi_m - a) + a \, \psi \right) - (\eta_0 + h)^2 \, \dot{\theta} \right] \right.$$

$$\left. + \frac{W_m}{g} \left[\epsilon \, \dot{\xi}_m \, f'(\xi_m - a) + a \, \dot{\psi} + \dot{\theta} \xi_m \right] \xi_m \right\}$$

$$= -W_m \cos \gamma \, (\xi_m - \xi_{mo}) - T(\eta_0 + h + \epsilon_2) + T \delta (\xi_m - a - d) - T \, \epsilon \, f(\xi_m)$$

$$\tag{4}$$

$$+ \frac{W_m}{g} \, \epsilon_1 \, \Omega^2 \sin(\Omega t + \lambda_1) \xi_m + I_{xy} \, \Omega^2 \sin(\Omega t + \lambda_2) - \beta \theta + T \, \psi \, \xi_m.$$

Although two more independent equations exist for the launcher, we shall not be concerned with them here since they only serve to define the unknown forces acting at the pivot point O.

For the model considered we shall employ Eqs. (1), (2), (3) and (4) to justify the approximations which will be presented in the next chapter.

II. APPROXIMATE ANALYSES

The inaccuracies described in Chapter I enter into the analytical description of the system through non-zero values of the parameters ϵ, ϵ_1, ϵ_2, δ, I_{xy}, μ. We disregard coupling effects and examine the consequences of taking these quantities to be non-zero one at a time.

Idealized Case (all parameters zero)

As our first problem, let us consider approximating the response of the ideal system: the one in which all of the inaccuracy parameters are zero. In this case Eqs. (1) and (4) become

$$\frac{W_m}{g} \left[\ddot{\xi}_m - (\eta_0 + h)\, \ddot{\theta} \right] = T - W_m \sin \gamma = T_e(t)\, T_0 \tag{5}$$

$$\frac{d}{dt} \left[\dot{\theta} (I_L + I_m + \frac{W_m}{g}\, \xi_m^2) \right] + \beta\, \theta = - (\xi_m - \xi_{mo})\, W_m \cos \gamma \tag{6}$$

We have neglected first order changes in moment arms for W_m and also assumed that β is very large. Neglecting the effects of θ in Eq. (5) we have

$$\xi_m = \frac{T_0\, g}{W_m} \int_0^t (t-\tau)\, T_e(\tau)\, d\tau + \xi_{mo}$$

and so Eq. (6) becomes

$$\frac{d}{dt} \left[\dot{\theta} (1 + \frac{W_m}{Ig}\, \xi_m^2) \right] + \omega^2 \theta = - \frac{T_0\, g}{I} \cos \gamma \int_0^t (t-\tau)\, T_e(\tau)\, d\tau \tag{7}$$

where $I = I_L + I_m$ and $\omega^2 = \beta/I$. Integrating this equation with respect to t, there results

$$\dot{\theta} (1 + \frac{W_m}{Ig}\, \xi_m^2) + \omega^2 \int_0^t \theta(\tau)\, d\tau = - \frac{T_0\, g \cos \gamma}{2I} \int_0^t (t-\tau)^2\, T_e(\tau)\, d\tau \tag{8}$$

We have used the fact that $\dot{\theta}(0) = 0$. This equation is essentially an impulse-momentum relation involving angular quantities. The first term is a change in angular momentum reduced by a factor I, and the remaining terms are angular impulses reduced by the same quantity. These impulses represent the effects of the spring β and the thrust. In order to estimate $\dot{\theta}$ at tip-off, it is necessary to know the dependence of θ on time up to tip-off.

Previous numerical studies along with field measurements indicate that most realistic systems have a θ-time history as shown in Fig. (5).
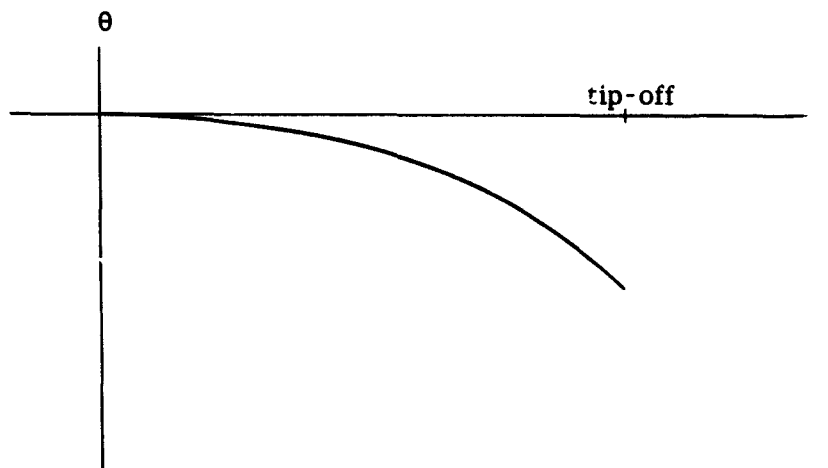

FIGURE 5

Since according to Eq. (7) and the initial conditions the second and third derivatives of θ vanish at $t = 0$, we shall sssume that

$$\theta = \frac{\dot{\theta}_p \, t^4}{4 \, t_p^3} \tag{9}$$

where $\dot{\theta}_p$ is the pitch velocity at tip-off time t_p. Placing Eq. (9) in Eq. (8) and performing the obvious reductions we find that

$$\dot{\theta}_p = \frac{-\dfrac{T_o\, g\, \cos\gamma}{2\,I} \displaystyle\int_0^t (t_p-\tau)^2\, T_e(\tau)\, d\tau}{1 + \dfrac{W_m}{g\,I}\, \xi_m^2(t_p) + \dfrac{(\omega\, t_p)^2}{20}} \quad. \tag{10}$$

For systems with large launch time $T_e(\tau)$ may be replaced by unity and in this case Eq. (10) simplifies to

$$\dot{\theta}_p = \frac{-\dfrac{T_o\, g\, \cos\gamma}{6\,I}\, t_p^3}{1 + \dfrac{W_m}{g\,I}\, \xi_m^2(t_p) + \dfrac{(\omega\, t_p)^2}{20}} \quad. \tag{11}$$

A comparison of this formula with known results indicates that it is fairly accurate when the time of launch t_p is one-half or less than the natural period of the system; that is,

$$t_p \leq \frac{1}{2}\, T_n = \frac{1}{2}\, \frac{2\pi}{\omega} \quad.$$

The reason for this can be obtained by converting Eq. (8), which we write more generally as

$$\dot{\theta}\, A(t) + \omega^2 \int_0^t \theta\, d\tau = F(t) \quad, \tag{12}$$

into an integral equation for $\dot{\theta}$ and then solving this equation by iteration. In Eq. (12)

$$A(t) = 1 + \frac{W_m}{g\,I}\, \xi_m^2(t)$$

and $F(t)$ is any forcing function. (As we shall see later most of the inaccuracies present themselves as different forcing functions.)

By noting that

$$\int_0^t \theta(\tau)\, d\tau = \int_0^t (t-\tau)\, \dot\theta(\tau)\, d\tau \qquad (13)$$

if $\theta(0) = \dot\theta(0) = 0$, we have immediately that Eq. (12) becomes

$$A(t)\, \dot\theta(t) + \omega^2 \int_0^t (t-\tau)\, \dot\theta(\tau)\, d\tau = F(t) \qquad (14)$$

which is now an integral equation in $\dot\theta$. If now we change variables according to

$$t = t_p\, u, \quad \frac{1}{(2\pi)^2} A(t) = B(u), \quad \frac{t_p}{(2\pi)^2} F(t) = G(u), \quad \frac{t_p}{T_n} = a, \qquad (15)$$

then we have

$$B(u)\, \frac{d\theta}{du} + a^2 \int_0^u (u-v)\, \frac{d\theta}{dv}\, dv = G(u). \qquad (16)$$

The solution of this equation can be expanded in a power series in a^2 according to

$$\frac{d\theta}{du} = \sum_{n=0}^{\infty} a^{2n}\, \frac{d\theta_{2n}}{du} \qquad (17)$$

where

$$B(u)\, \frac{d\theta_0}{du} = G(u)$$

and

$$B(u)\, \frac{d\theta_{2n+2}}{du} = -\int_0^u (u-v)\, \frac{d\theta_{2n}}{dv}\, dv.$$

In particular taking $T_e(\tau) = 1$ we find

$$\theta'_0(u) = -\frac{T_o \, g \cos \gamma \, t_p^4 \, u^3}{6 \, I \, A(u)} \quad , \qquad \theta'_2(u) = \frac{T_o \, g \cos \gamma \, t_p^4}{6 \, I \, A(u)} \int_0^u \frac{(u-v) \, v^3}{B(v)} \, dv$$

so that

$$\dot{\theta}(t_p) = -\frac{T_o \, g \cos \gamma \, t_p^3}{6 \, I \, A(t_p)} \left[1 - a^2 \int_0^1 \frac{(1-v) \, v^3 \, dv}{B(v)} \right]$$

$$+ \text{Error } O(a^4 \dot{\theta}_4) \, . \tag{18}$$

When a is sufficiently small the first term in Eq. (18) gives $\dot{\theta}(t_p)$ fairly accurately. Since

$$\int_0^1 \frac{(1-v) \, v^3 \, dv}{B(v)} \geqslant \frac{1}{20 \, B(1)}$$

we can write

$$\dot{\theta}(t_p) = -\frac{T_o \, g \cos \gamma \, t_p^3}{6 \, I \, A(t_p)} \left[1 - \frac{a^2}{20 \, B(1)} \right]$$

$$\doteq -\frac{T_o \, g \cos \gamma \, t_p^3}{6 \, I} \quad \frac{1}{A(t_p) \left[1 + \frac{a^2}{20 \, B(1)} \right]}$$

$$= \frac{-\dfrac{T_o \, g \cos \gamma \, t_p^3}{6 \, I}}{1 + \dfrac{W_m}{g \, I} \, \xi_m^2(t_p) + \dfrac{(\omega \, t_p)^2}{20}}$$

when $a^2 / 20 \, B(1)$ is small. But this expression is identical with Eq. (11).

$$\frac{d}{dt}\left[\dot{\Delta}_T \ A(t)\right] + \omega^2 \, \Delta_T = -\frac{T \, \delta \, (a+d)}{I} + \frac{T \, \delta \, \xi_m}{I} \ . \tag{20}$$

Eq. (20) is the differential equation satisfied by the variation in pitch velocity due to thrust misalignment. This is readily converted into an integral equation in $\dot{\Delta}_T$:

$$\dot{\Delta}_T \ A(t) + \omega^2 \int_0^t (t-\tau) \, \dot{\Delta}_T (\tau) \, d\tau = \int_0^t \left[-\frac{T \, \delta \, (a+d)}{I} + \frac{T \, \delta \, \xi_m}{I}\right] d\tau. \tag{21}$$

When t_p/T_n is small two iterations are sufficient for a solution and we find that

$$\dot{\Delta}_T(t_p) \ A(t_p) \doteq \frac{T_o \, \delta}{I} \left[\frac{T_o \, g}{6 \, W_m} \ t_p^3 + (\xi_{mo} - a - d) \, t_p\right]$$

$$- \frac{\omega^2 \, T_o \, \delta}{A(t_p)} \left[\frac{T_o \, g}{6 \, W_m \, I} \ \frac{t_p^5}{20} + \frac{(\xi_{mo} - a - d)}{6 \, I} \ t_p^3\right] \tag{22}$$

for the case of constant thrust.

For those cases in which this two term approximation is not valid a solution of Eq. (21) based on the process of Eq. (19) is in order.

Shaking Forces

When the missile spins on the rail the equation for the deviation in pitch velocity $\Delta_s = \theta_s - \theta$ is of the form

$$\frac{d}{dt}\left[A(t) \ \dot{\Delta}_s(t)\right] + \omega^2 \, \Delta_s = \frac{I_{xy} \, \Omega^2}{I} \ \sin(\Omega \, t + \lambda_1)$$

$$+ \frac{W_m}{g \, I} \ \epsilon_1 \, \Omega^2 \, \xi_m \ \sin(\Omega \, t + \lambda_2) \tag{23}$$

where λ_1, λ_2 are random angles between 0 and 2π. This is again reduced to

an integral equation in $\dot{\Delta}_s$:

$$A(t) \dot{\Delta}_s + \omega^2 \int_0^t (t-\tau) \dot{\Delta}_s \, d\tau = \dot{\Delta}_s(0)$$

$$+ \frac{I_{xy} \Omega}{I} \left[\cos \lambda_1 - \cos(\Omega t + \lambda_1) \right] + \frac{W_m}{I g} \epsilon_1 \Omega^2 \int_0^t \xi(\tau) \sin(\Omega \tau + \lambda_2) \, d\tau \tag{24}$$

The solution of this equation by iteration yields very unwieldy expressions. Furthermore because of the random character of $\dot{\Delta}_s(0)$, λ_1, λ_2 $\dot{\Delta}_s$ in itself is indeterminate. It is more appropriate to speak of the mean value of $\dot{\Delta}_s$, $\overline{\Delta}_s$, and its standard deviation $\sigma \left[\dot{\Delta}_s \right]$. These can be obtained by first noting that Eq. (24) can be written in the form

$$A(t) \dot{\Delta}_s + \omega^2 \int_0^t (t-\tau) \dot{\Delta}_s \, d\tau = \dot{\Delta}_s(0) L_0(t) + L_1(t) \sin \lambda_1 + L_2(t) \cos \lambda_1$$

$$+ L_3(t) \sin \lambda_2 + L_4(t) \cos \lambda_2$$

where $L_0(t) \equiv 1$. Writing

$$\dot{\Delta}_s(t) = \dot{\Delta}_s(0) \dot{\Delta}_{so}(t) + \dot{\Delta}_{s1}(t) \sin \lambda_1 + \dot{\Delta}_{s2}(t) \cos \lambda_1 + \dot{\Delta}_{s3}(t) \sin \lambda_2$$

$$+ \dot{\Delta}_{s4}(t) \cos \lambda_2 \tag{25}$$

we find that

$$A(t) \dot{\Delta}_{si}(t) + \omega^2 \int_0^t (t-\tau) \dot{\Delta}_{si} \, d\tau = L_i(t) \tag{26}$$

Each of these can be solved for $\dot{\Delta}_{si}(t_p)$ by the process outlined in Eq. (19). Consequently the mean value and standard deviation of $\dot{\Delta}_s(t_p)$ due to the initial value $\dot{\Delta}_s(0)$ is

$$\overline{\left[\dot{\Delta}_s(t_p)\right]}_{\text{initial value}} = \left[\dot{\Delta}_s(0)\right] \dot{\Delta}_{so}(t_p)$$

$$\sigma\left[\dot{\Delta}_s(t_p)\right]_{\text{initial value}} = \sigma\left[\dot{\Delta}_s(0)\right] \dot{\Delta}_{so}(t_p)$$

and for the parameter λ_1

$$\overline{\left[\dot{\Delta}_s(t_p)\right]}_{\lambda_1} = 0 \quad , \qquad \sigma\left[\dot{\Delta}_s(t_p)\right]_{\lambda_1} = \sqrt{\frac{\dot{\Delta}_{s1}^2(t_p) + \dot{\Delta}_{s2}^2(t_p)}{2}}$$

and for the parameter λ_2

$$\overline{\left[\dot{\Delta}_s(t_p)\right]}_{\lambda_2} = 0, \qquad \sigma\left[\dot{\Delta}_s(t_p)\right]_{\lambda_2} = \sqrt{\frac{\dot{\Delta}_{s3}^2(t_p) + \dot{\Delta}_{s4}^2(t_p)}{2}}$$

Consequently the expected mean value of $\dot{\Delta}_s(t_p)$ is

$$\overline{\dot{\Delta}_s(t_p)} = \left[\dot{\Delta}_s(0)\right] \dot{\Delta}_{so}(t_p)$$

and the standard deviation is

$$\sigma\left[\dot{\Delta}_s(t_p)\right] = \sqrt{\sigma^2\left[\dot{\Delta}_s(0)\right] \dot{\Delta}_{so}^2(t_p) + \frac{1}{2}\left[\dot{\Delta}_{s1}^2(t_p) + \dot{\Delta}_{s2}^2(t_p) + \dot{\Delta}_{s3}^2(t_p) + \dot{\Delta}_{s4}^2(t_p)\right]}.$$

In computing these we have assumed that λ_1, λ_2, $\dot{\Delta}_s(0)$ are independent.

Friction

In treating friction we assume that ξ_m is given by the expression

$$\xi_m = \frac{T_o\, g}{W_m} \int_0^t (t-\tau)\, T_e(\tau)\, d\tau + \xi_{mo}$$

and write

$$\frac{d}{dt}\, \theta_f\, A(t) + \omega^2\, \theta_f = -\mu\, \frac{|N|}{I}(\eta_0 + h) - (\xi_m - \xi_{mo})\frac{W_m}{I}\cos\gamma \qquad (27)$$

where N is taken as

$$N = W_m \cos \gamma + \frac{W_m}{g} (\xi_m \ddot{\theta}_f + \theta \dot{\xi}_m \dot{\theta}_f).$$

So long as N > 0 we may write

$$\dot{\theta}_f \left[A(t) + \mu \frac{W_m}{g} \frac{(\eta_0 + h)}{I} \xi_m \right] + \int_0^t \left[\omega^2 (t-\tau) + \mu \frac{(\eta_0 + h)}{I} \frac{W_m}{g} \dot{\xi}_m \right] \dot{\theta}_f (\tau) \, d\tau$$

$$= - \frac{T_0 \, g \cos \gamma}{6 \, I} t^3 - \frac{1}{I} \mu (\eta_0 + h) W_m \cos \gamma \quad t. \tag{28}$$

An approximate expression for $\dot{\theta}_f$ follows by iteration. When this is done there results

$$\dot{\theta}_f (t_p) = - \left[1 + \frac{W_m}{g I} \xi_m^2 (t_p) + \frac{\mu W_m}{g} \frac{(\eta_0 + h)}{I} \xi_m (t_p) \right]^{-1}$$

$$\left\{ \frac{T_0 \, g \cos \gamma}{6 \, I} t_p^3 + \frac{\mu (\eta_0 + h)}{I} W_m \cos \gamma \, t_p \right.$$

$$- \frac{T_0 \, g \cos \gamma}{6 \, I} \frac{t_p^5}{20} \left[\omega^2 + \frac{4 \mu (\eta_0 + h)}{I} T_0 \right]$$

$$\left. - \frac{\mu (\eta_0 + h)}{I} W_m \cos \gamma \frac{t_p^3}{6} \left[\omega^2 + \frac{2 \mu (\eta_0 + h)}{I} T_0 \right] \right\}$$

A more accurate solution can be obtained by writing Eq. (28) as

$$C(t) \dot{\theta}_f + \int_0^t D(t, \tau) \dot{\theta}_f (\tau) \, d\tau = E(t) \tag{29}$$

and solving Eq. (29) numerically. By converting to the variable $t = t_p u$ we have

$$L(u) \theta'_f (u) + \int_0^u M(u, v) \frac{d\theta}{dv} \, dv = N(u)$$

Splitting the range (0, 1) into n equal subintervals there results

$$\left[L\left(\frac{k}{n}\right) + M\left(\frac{k}{n}, \frac{k}{n}\right) \right] \theta'_f\left(\frac{k}{n}\right) + \sum_{m=1}^{k-1} M\left(\frac{k}{n}, \frac{m}{n}\right) \theta'_f\left(\frac{m}{n}\right) = N\left(\frac{m}{n}\right)$$

$$\theta'_f(0) = 0 \quad , \qquad \theta'_f\left(\frac{1}{n}\right) = \frac{N(1/n)}{L\left(\frac{1}{n}\right) + M\left(\frac{1}{n}, \frac{1}{n}\right)} \quad .$$

Rail Curvature

As a final item we consider rail curvature. In this case all we present is an integral equation. Any solution of this equation hinges on an assumed form for rail curvature. If Δ_R denotes the difference $\theta_R - \theta$ where θ_R is the angular response in the presence of rail curvature Eq. (4) reduces to

$$\frac{d}{dt} A(t) \dot{\Delta}_R + \omega^2 \Delta_R = \frac{W_m}{Ig} \frac{d}{dt} \left[\epsilon \dot{\xi}_m f(\xi_m - a) + a \psi \dot{\xi}_m - \epsilon \dot{\xi}_m \xi_m f'(\xi_m - a) \right.$$

$$\left. - a \dot{\psi} \xi_m \right] - \frac{W_m \cos \gamma}{I(a+b)} \epsilon \left[b f'(\xi_m - a) + a f'(\xi_m + b) \right]$$

$$- \frac{T \epsilon}{I} f(\xi_m) + \frac{T}{I} \xi_m \psi \quad . \tag{30}$$

It is worth noticing that rail curvature introduces a thrust misalignment in a sense. The appropriate integral equation is

$$A(t) \dot{\Delta}_R + \omega^2 \int_0^t (t-\tau) \dot{\Delta}_R \, d\tau = \frac{W_m}{Ig} \left[\epsilon \dot{\xi}_m f(\xi_m - a) - \epsilon \dot{\xi}_m \xi_m f'(\xi_m - a) \right.$$

$$\left. + a \psi \dot{\xi}_m - a \dot{\psi} \xi_m \right]$$

$$- \frac{1}{I} \int_0^t \left\{ \frac{W_m \cos \gamma}{a+b} \epsilon \left[b f'(\xi_m - a) + a f'(\xi_m + b) \right] + T \epsilon f(\xi_m) - T \xi_m \psi \right\} d\tau.$$

This is, again of the form which Eq. (19) solves numerically.

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